Nonlinear Piezo-Actuator Control by Learning Self-Tuning Regulator

C. James Li,[1] Homayoon S. M. Beigi,[2]
Shengyi Li,[3] and Jiancheng Liang[3]

This paper presents a learning self-tuning (LSTR) regulator which improves the tracking performance of itself while performing repetitive tasks. The controller is a self-tuning regulator based on learning parameter estimation. Experimentally, the controller was used to control the movement of a nonlinear piezoelectric actuator which is a part of the tool positioning system for a diamond turning lathe. Experimental results show that the controller is able to reduce the tracking error through the repetition of the task.

Nomenclature

- α = output of the plant
- γ = forgetting factor in the repetition domain
- λ = forgetting factor in the time domain
- ϕ = data vector
- θ = parameter vector
- A = polynomial of the system poles
- B = polynomial of the system zeroes
- $\mathbf{C}$ = vector of generalized forces due to centrifugal and Coriolis forces
- $\mathbf{c}$ = damping constant
- $\mathbf{F}$ = vector of generalized viscous friction forces
- $\mathbf{G}$ = vector of generalized gravitational forces

[1]Department of Mechanical Engineering, Columbia University, New York, NY 10027.

[2]International Business Machines, T. J. Watson Research Center, Yorktown Heights, NY 10598.

[3]Department of Precision Machinery and Instruments, Changsha Institute of Technology, Changsha, Hunan, PRC.

Contributed by the Dynamic Systems and Control Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received by the DSCD May 1990; revised manuscript received December 1992. Associate Technical Editor: H. Kazerooni.

I = identity matrix
k = time step
P = covariance matrix of errors in parameter estimates
p = no. of sampling intervals for the trajectory
q^{-d} = d step delay operator
r = repetition number
t = time step
u = control actions (plant input)
v = equation error

I Introduction

Several advanced schemes have been proposed for an improved performance, which would generate control actions to accommodate nonlinear dynamics. These include, for example, nonlinear feedback control [1], feedforward control [2], resolved motion control [3, 4], sliding mode control [5], repetitive control [6, 7], and learning control [8]. In particular, adaptive controls have drawn a lot of attention in various applications [9–12]. One class of these adaptive controllers is the self-tuning regulator [9].

A self-tuning regulator consists of a parameter estimator and a controller. The parameter estimator estimates the parameters of an approximated model of the controlled system by utilizing a recursive estimation scheme. Based on the approximate model and the estimated parameters, the controller adjusts its actions to maintain its performance. Therefore, the performance of a self-tuning regulator depends greatly on that of the parameter estimator employed.

The existing recursive least square (RLS) algorithms do not utilize their past experience in estimating repetitive parameters. The implication is that the system will keep making the same errors at corresponding times in the duration of each repetition.

Another limit of these estimators is that they have to keep the changes of estimations small between neighboring sampling instants to maintain their immunity to noise and disturbances. Consequently, the estimators cannot respond to large changes of parameters quickly.

The objective of this paper is to present a learning adaptive control scheme based on a learning estimator which utilizes the information from past performances of a repetitive task to refine the estimate of the plant parameters. Consequently, the learning adaptive controller improves its performance throughout the repetitions. Since the adaptation of parameters at all sampling instants is made over the repetitions of the task, the estimator is free to follow the changes of parameters over time.

II The Self-Tuning Regulator

References [12–14] provide a detailed description of the self-turning regulator and parameter estimation. The self-turning regulator has been applied to the control of nonlinear systems such as robots based on linear discrete uncoupled models of the plant. The rationale of this approach is that linearization of the nonlinear governing equations at each sampling instants would yield linear difference equations of time-varying parameters.

Typically, a self-tuning regulator models a plant with a difference equation such as follows and uses a recursive estimator to estimate the parameter vector $\theta(t)$ at each sampling instant and adjusts its control action accordingly. Therefore, the overall performance of the control system is very much dependent on how close these estimates are to the real system parameters. The difference equation has the form of:

$$\alpha(k) = \phi'(k)\theta(k) + v(k) \tag{1}$$

where

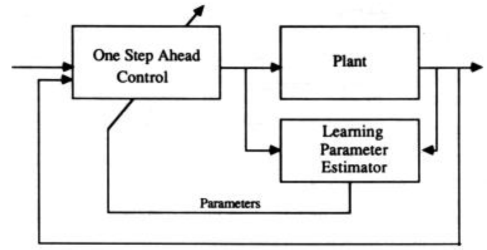

Fig. 1 The learning self-tuning regulator

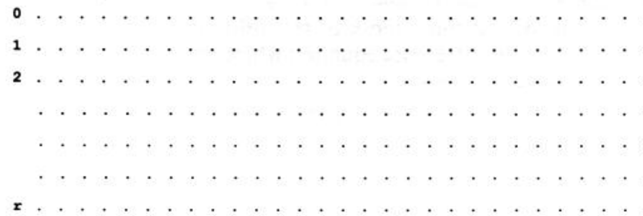

Fig. 2 Sampling instants in each repetition of a task

$$\theta'(k) = [\theta_1(k), \theta_2(k), \theta_3(k), \theta_4(k), \ldots, \theta_{N+M}(k)],$$
$$\phi'(k) = [-\alpha(k-1), -\alpha(k-2), \ldots,$$
$$-\alpha(k-N), u(k-1), u(k-2), \ldots, u(k-M)].$$

To find a θ vector which would minimize

$$v_T(\theta) = \sum_{k=0}^{T} \lambda^{T-k}[\alpha(k) - \theta'(k)\phi(k)]^2 \tag{2}$$

the following algorithm can be used [13, 14]:

$$\theta(k) = \theta(k-1) + \mathbf{L}(k-1)[\alpha(k) - \theta'(k-1)\phi(k)] \tag{3a}$$

$$\mathbf{L}(k-1) = \frac{\mathbf{P}(k-1)\phi(k)}{\lambda + \phi'(k)\mathbf{P}(k-1)\phi(k)} \tag{3b}$$

$$P(k) = \frac{[I - L(k-1)\phi'(k)]P(k-1)}{\lambda} \tag{3c}$$

Typically, an initial estimate of the parameters has to be given to the estimator to start the recursion. Frequently, this estimate is different from the real parameters. Therefore, a tracking error will be produced until the parameter estimator catches up which may never be achieved if the parameters are varying from one sampling instant to the next. Consequently, a persistent tracking error resulted from the lagging of the estimate of the parameters at all times is inevitable.

If the same task is executed repeatedly, the controller repeats it without looking back at how it had performed previously. Therefore, the controller will make the same errors at corresponding times of repetitions.

The following proposition of a learning estimator provides a solution which will allow the use of the information acquired from previous repetitions to improve the estimate of the parameters over repetitions of a task. With better estimates of parameters the self-turning controller will be able to reduce tracking errors over the repetitions of a task.

III Learning Recursive Least Squares Estimator and the Control Law

Figure 1 illustrates the complete closed-loop system which consists of the learning parameter estimator and the One-Step-Ahead controller which was selected for its simplicity [9].

We will use Fig. 2 to illustrate the idea behind the proposed learning parameter estimator. Each dot in Fig. 2 represents

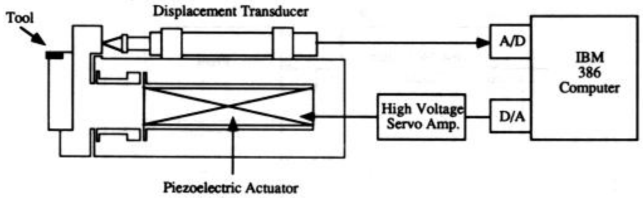

Fig. 3 The experimental setup of the diamond turning tool

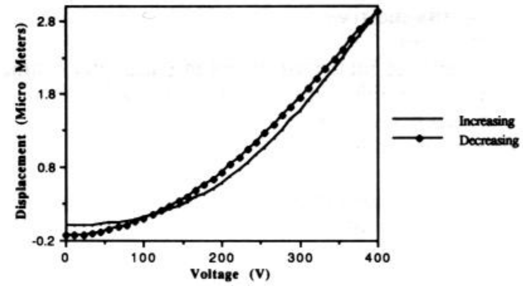

Fig. 4 Measured static response of the piezoelectric tool positioning system

one sampling instant. Each row of $(p + 1)$ dots represents a run of the task which is numbered from repetition 0 to repetition r.

Let us look at time step k of repetition r, and assume that there is a linear difference equation which adequately describes the dynamics of the plant.

$$\alpha^r(k) = \underline{\phi}^{r'}(k)\underline{\theta}^r(k) + v^r(k) \tag{4}$$

where,

$$\underline{\theta}^{r'}(k) = [\theta_1^r(k), \theta_2^r(k), \theta_3^r(k), \theta_4^r(k), \ldots, \theta_{N+M}^r(k)],$$

$$\underline{\phi}^{r'}(k) = [-\alpha^r(k-1), -\alpha^r(k-2), \ldots, -\alpha^r(k-N), u^r(k-1), u^r(k-2), \ldots, u^r(k-M)]$$

For $r = 0, 1, 2, \ldots, r$, Eq. (4) gives a set of linear algebraic equations which may be written in the matrix form as

$$\underline{\alpha}^r(k) = \Phi^r(k)\underline{\theta}^r(k) + \underline{v}^r(k)$$

where,

$$\underline{v}^{r'}(k) = [v^0(k), v^1(k), v^2(k), \ldots, v^r(k)]$$

$$\underline{\alpha}^{r'}(k) = [\alpha^0(k), \alpha^1(k), \alpha^2(k), \ldots, \alpha^r(k)] \tag{5}$$

and

$$\Phi^{r'}(k) = [\underline{\phi}^0 \quad \underline{\phi}^1 \quad \ldots \quad \underline{\phi}^r]$$

It can be easily shown that the least square estimate of $\underline{\theta}^r(k)$ minimizing the sum of the squares of the equation errors, $v^r(k)s$ can be calculated recursively as follows [15]:

$$\underline{\hat{\theta}}^{r+1}(k) = \underline{\hat{\theta}}^r(k) + \underline{L}^r(k)[\alpha^{r+1}(k) - \underline{\hat{\theta}}^{r'}(k)\underline{\phi}^{r+1}(k)] \tag{7}$$

$$\underline{L}^r(k) = \frac{P^r(k)\underline{\phi}^{r+1}(k)}{\gamma + \underline{\phi}^{r+1'}(k)P^r(k)\underline{\phi}^{r+1}(k)} \tag{8}$$

$$P^{r+1}(k) = \frac{[I - \underline{L}^r(k)\underline{\phi}^{r+1'}(k)]P^r(k)}{\gamma} \tag{9}$$

For a repetitive task which takes p sampling interval (p + 1 sampling instants) to complete, one would need $(p + 1)$ linear difference equations to approximate the real equation of motions. The $(p + 1)$ sets of parameters of these difference equations can be updated independently by Eq. (7) from one repetition to the next. Since this updating does not have to be done on-line as long as it is done before the next repetition is started, the demand on the on-line computing power is much less than traditional adaptive controllers.

V Experimental Results

In order to study the behavior of the proposed controller in the control of a real nonlinear plant, the controller was used to control a piezoelectric actuator which is part of a diamond turning lathe. In precision machining such as diamond turning, there is a lot of repetitive error sources such as spindle run-off error and guideway geometric error which would produce inaccuracies on the workpiece if left uncompensated for. Thus, a tool positioning system is called for.

The experimental setup is illustrated in Fig. 3. The tool holder, which is made out of a complete piece of steel using an electric discharge machine, has a construction of two parallel elastic plates which provides a small axial rigidity in com-

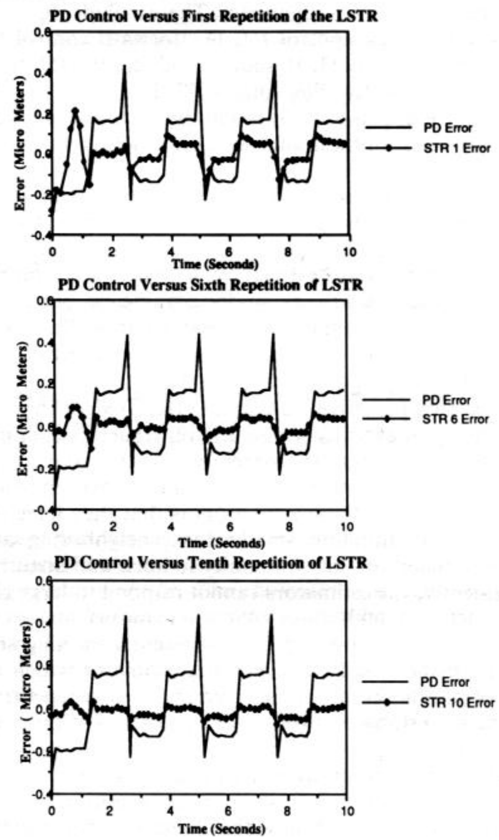

Fig. 5 Tracking error of the piezo tool at repetitions 0, 1, 6 and 10

parison to other degrees of freedom. Therefore, the diamond cutting tool moves back and forth as the piezoelectric actuator expands and shrinks. This movement is measured by the inductive displacement sensor which is an integral part of the tool holder.

The piezoelectric actuator is made of PZT-La stack of fifty disks which has a dimension of 14 × 16 × 0.5 mm. The actuator generates displacement through the mechanism of electrostrictive. This material has less hysteresis loss and better aging property than that of piezoelectric material. However, it has a very nonlinear characteristic between the displacement and the electrical potential applied to it. The parabolic characteristic curve of the actuator is shown in Fig. 4. The presence of the hysteresis nonlinearity is also obvious since the curve follows different paths while the voltage is increased and decreased.

In our experiments, the tool is commanded to follow a square wave with an amplitude of 1.1 μm and a period of 2.3435 second. The sampling period is 0.15625. The first execution

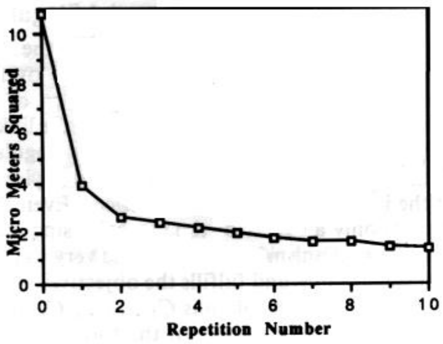

Fig. 6 Sum of error squares over repetitions for piezo tool control

of the task was carried out by a PD controller. Then, the learning adaptive controller performed the following repetitions. At the beginning, we used a second order difference equation as the plant model and the tracking error actually got larger and larger over the repetitions. Thus, we increase the order of the model from two to three. This enabled us to obtain a satisfactory controller.

Figure 5 shows the tracking error versus time measured by the inductive position transducer at repetition 0, 1, 6 and 10. It is obvious that significant reduction of tracking errors was achieved by the proposed controller over the repetition of the desired trajectory. Figure 6 shows that the sum of squares of errors decreases as the number of repetitions increases.

VI Conclusion and Future Development

Experiment results show that the learning self-tuning regulator is a quick learning controller which reduces tracking errors of the nonlinear system over repetitions. Also, the fact that the parameter updating part could be done off-line between repetitions, makes this control algorithm very feasible. In fact, compared with a PD (proportional plus derivative) controller, only three more additions and three more multiplications have to be done in real-time at each time step if a second order difference equation is used as the model. However, more memory is needed to implement the controller. The amount of memory needed for implementing the controller for an n degree of freedom system is $= 22 * \#$ of time steps in each repetition $* n$.

The experimental results have shown that the proposed learning self-tuning regulator can be applied to the control of a nonlinear system such as a piezo tool positioning system. Possible applications include the control of robots, precision machining, process controls and other manufacturing processes. Also, the notion of a learning parameter estimator could be applied to other parameter estimators than the RLS estimator.

References

1 Bejczy, A. K., Tarn, T. J., and Chen, Y. L., "Computer Control of Robot Arms," *Proceedings of First IEEE International Conference on Robotics and Automation*, St. Louis, MO, Mar. 1985.

2 Craig, J. J., *Introduction to Robotics Mechanics and Control*, Addison-Wesley, Reading, MA, 1986.

3 Whitney, D. E., "Resolved Motion Rate Control of Manipulators and Human Prostheses," *IEEE Transactions of Man, Machines, and Systems*, Vol. MMS-10, No. 2, June 1969, pp. 47–53.

4 Luh, J. Y. S., Walker, M. W., and Paul, R. P., "Resolved Acceleration Control of Mechanical Manipulators," *IEEE Transactions on Automatic Control*, Vol. AC-25, No. 3, June 1980, pp 468–474.

5 Slotine, J. E., "Sliding Controller Design for Non-linear Systems," *International Journal of Control*, Vol. 40, No. 2, 1984, pp. 421–434.

6 Middleton, R. H., Goodwin, G. C., and Longman, R. W., "A Method for Improving the Dynamic Accuracy of a Robot Performing a Repetitive Task," *International Journal of Robotic Research*, Vol. 8, No. 5, Oct. 1988, pp. 67–74..

7 Tomizuka, M., Tsao, T. C., and Chew, K. K., "Discrete-Time Domain Analysis and Synthesis of Repetitive Controllers, *Proc. of American Control Conference*, Atlanta, GA, June 1988, pp. 860–866.

8 Kawamura, S., Miyazaki, F., and Arimoto, S., "Applications of Learning Methods for Dynamic Controls of Robot Manipulators," *Proceedings of the 24th IEEE CDC*, Fort Lauderdale, Florida, Dec. 1985.

9 Goodwin, G. C., and Sin, K. S., *Adaptive Filtering Prediction and Control*, Prentice-Hall, NJ, 1984.

10 Dubowksy, S., and DeForges, D. T., "The Application of Model Referenced Adaptive Control to Robot Manipulators," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Vol. 101, pp. 193–200, Sept. 1979.

11 Craig, J. J., *Adaptive Control of Mechanical Manipulators*, Addison-Wesley Publishing Company, New York, 1987.

12 Åström, K. J., "Adaptive Feedback Control," *Proceedings of the IEEE*, Vol. 75, No. 2, Feb. 1987.

13 Åström, K. J., and Wittenmark, B., *Computer Controlled Systems Theory and Design*, Prentice-Hall, NJ, 1984.

14 Ljung, L., and Sodestrom, T., *Theory and Practice of Recursive Identification*, The MIT Press, Cambridge, MA, 1986.

15 Li, C. James, and Beigi, Homayoon S. M., "A Self-tuning Regulator with Learning Parameter Estimation," *Proc. of the Symposium of Robotics*, 1990 ASME WAM, Dallas, Texas, Dec. 1990, pp. 1–6.